III. Solution by the PROPOSER.

The foci S, S' of (A), $y^2=4ax$, and (B) $x^2=4by$, are, respectively, $(a, 0)$, $(0, b)$, and the circle in question has as its equation $x^2+y^2-ax-by=0$.

Since the common vertex V is $(0, 0)$ and S' is $(0, b)$, the point M in which DE cuts the axis of B is $0, 2b$, and the line DE is $y=2b$.

This cuts the circle where $x^2-ax+2b^2=0$...(1), giving D and E. If the roots of this be x_1 and x_2, D, E are $(x_1, 2b)$, $(x_2, 2b)$, and $S'D$, $S'E$ have respectively, for their equations,

$$bx-x_1y+x_1b=0 \text{ and } bx-x_2y+x_2b=0.$$

The tangent at $(at^2, 2at)$ to A is $x-ty+at^2=0$.
The normal at $(2bt', at'^2)$ to B is $x+t'y-2bt'-bt'^3=0$.
These are the same line if $1=\dfrac{-t}{t'}=\dfrac{-at^2}{bt'(2+t'^2)}$.
$\therefore t=-t'$, and rejecting the values $t=t'=0$ we have $t^2b-at+2b=0$...(2).
The roots of (1) and (2) are the same, and the property is proved.
Perhaps Prof. Zerr will reconsider the solution he offers.

CALCULUS.

263. Proposed by V. M. SPUNAR, M. S., C. E., East Pittsburg, Pa.

Find a point such that the sum of the squares of its distances from n given points shall be a minimum, and prove that the value so found is $1/n$th part of the sum of the squares of the mutual distances between the n points, taken two and two.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

This problem is proposed and solved in Williamson's *Differential Calculus*, 9th edition, page 192, Art. 157.

Taking one of the points as origin, and the axes rectangular, let (x, y) be the coordinates of the required point. Let (a_1, b_1); (a_2, b_2); (a_3, b_3); ..., (a_{n-1}, b_{n-1}) be the coordinates of the other $(n-1)$ points. Then

$$x^2+y^2+(x-a_1)^2+(y-b_1)^2+(x-a_2)^2+(y-b_2)^2+\dots$$
$$+(x-a_{n-1})^2+(y-b_{n-1})^2=u=\text{minimum, or}$$

$$nx^2+ny^2-2(a_1+a_2+\dots+a_{n-1})x-2(b_1+b_2+\dots+b_{n-1})y$$
$$+a_1^2+a_2^2+\dots+a^2_{n-1}+b_1^2+b_2^2+\dots+b^2_{n-1}=u=\text{minimum.}$$

$$\therefore (nx-a_1-a_2-\dots-a_{n-1})dx+(ny-b_1-b_2-\dots-b_{n-1})dy=0.$$

$$\therefore x=\frac{a_1+a_2+\dots+a_{n-1}}{n},\ y=\frac{b_1+b_2+\dots+b_{n-1}}{n}.$$

The point is, therefore, the center of mean position of the n points as stated.

Also solved by J. Scheffer.

264. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

The join of the center of curvature of a curve to the origin is at α to the initial line. Prove that with the usual notation:

$$\frac{d\alpha}{d\psi}\left[\left(\frac{dp}{d\psi}\right)^2+\left(\frac{d^2p}{d\psi^2}\right)^2\right]=\frac{dp}{d\psi}\cdot\frac{d\rho}{d\psi}.$$

No solution of this problem has been received.

265. Proposed by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Find two curves which possess the property that the tangents TP and TQ to the inner one always make equal angles with the tangent TT' to the outer.

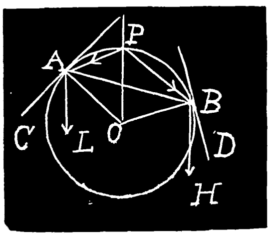
No solution of this problem has been received.

MECHANICS.

219. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

A rod length $a\sqrt{3}$, weight W, has at each end a smooth ring which can slide on a vertical circle radius r. Each ring is attached by an elastic string (natural lengths a, b; moduli μa, μb) to the highest point of the circle. Find the inclination of the rod to the horizon in a position of equilibrium.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.



In what follows we regard the strings as having no weight, and also that both strings are in tension from the weight of the rod and rings, and the rod is above the center of the circle. Let $AB=a\sqrt{3}=$rod; $AP=$string a; $BP=$string b; O, the center of the circle, radius $AO=r$. Draw AK perpendicular to PO. Let $m=$ weight of each ring; $\angle AOB=\beta=2\sin^{-1}(a\sqrt{3}/2r)$; $\angle APB=\pi-\frac{1}{2}\beta$; $\angle KAB=\theta=$angle AB makes with the horizon; $\angle PAB=\phi$; $\angle PBA=\frac{1}{2}\beta-\phi$; $T=$tension of string AP; $T'=$tension of string PB. When in equilibrium, W and the components of T, T' tangent at A, B meet in a point. $AP=a(1+T/\mu a)=(\mu a+T)/\mu$, $BP=(\mu b+T')/\mu$, $T=(\frac{1}{2}W+m)\sin(\phi-\theta)$. Let $(\frac{1}{2}W+m)=Q$.

$\therefore T=Q\sin(\phi-\theta)$, $T'=Q\sin(\theta+\frac{1}{2}\beta-\phi)$, $AP/PB=(\mu a+T)/(\mu b+T')=\sin(\frac{1}{2}\beta-\phi)/\sin\phi$... (1).

$3\mu^2a^2=(\mu a+T)^2+(\mu b+T')^2+2(\mu a+T)(\mu b+T')\cos\frac{1}{2}\beta$... (2).

The values of T and T' in (1) and (2) give

$[\mu a+Q\sin(\phi-\theta)]\sin\phi=[\mu b+Q\sin(\theta+\frac{1}{2}\beta-\phi)]\sin(\frac{1}{2}\beta-\phi)$... (3).